A METHOD FOR THE DETERMINATION OF ALL THE PROPAGATING MODES IN A LOADED WAVEGUIDE STRUCTURE

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Abstract

A method has been investigated for the accurate determination of the total number of propagating modes, the electromagnetic field patterns and the power density distribution in a transversely magnetized, ferrite loaded rectangular waveguide.

The established criterion is valid for any ferrite material thickness and gives more complete results than the first method developped by Gardiol et al.

Introduction

Waveguides loaded with slabs, ridges or rods of magnetized ferrites, semiconductors, dielectrics and/or absorbing materials have found numerous uses in the development of microwave devices. For practical reasons, only the frequency range over which a single mode can propagate is usable, in order to avoid the attenuation and mismatch spikes (moding) due to the occurrence of propagating higher-order modes. Therefore, a certain knowledge of the propagation characteristics of all the modes which can exist in a given structure is necessary to the designer. The first higher-order mode, i.e. the mode having the second lowest cutoff frequency sets the upper bound to the frequency band that can safely be used in actual operation (unless suitable precautions are taken to prevent excitation of this mode).

Higher-order mode characteristics are difficult to determine experimentally, due to the presence of the propagating dominant mode. Extrapolation from the empty waveguide is often misleading; in rectangular waveguides for instance, the assumption that the first higher-order mode is the TE_{20} mode, which is the case in the empty waveguide, was shown to be quite erroneous in most cases when slabs of dielectric are introduced in the guide[1]. It becomes, therefore, necessary to determine theoretically the propagation characteristics of the structure. In some cases, an exact analytical solution can be obtained [2-3] while for other structures, only approximations are available, such as the Rayleigh-Ritz method[4]

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Position Problem

A feature which is common to most of these methods is that they lead to a characteristic or determinantal equation. The propagation coefficient is obtained by solving an equation having the general form :

$$f(\gamma) = 0 , \qquad \gamma = \alpha + j.\beta \tag{1}$$

This equation is the determinant of the boundary-value equations in exact analytical methods; in this case, the function is transcendental, involving trigonometric functions in rectangular waveguides, Bessel functions in circular, Mathieu functions in elliptical, etc... For the Rayleigh-Ritz approximation, the function is given by the determinant of a set of linear equations and could thus be solved exactly. However, since a large number of terms of the expansion must be taken to yield adequate accuracy [5] a complete solution would be quite time-consuming and unnecessary. When suitable precautions are taken during the derivation of (1), the function F (γ) is analytical; its zeros located at the points γ_k in the complex γ plane corresponding to the modes of the structure. Dividing by any function which has zeros in the γ plane (for simplification reasons) is not allowed as it would introduce singularities[1]. For a lossless system, the zeros corresponding to propagating modes are all located on the imaginary axis of the γ plane. This is no longer true for lossy structures, for which the γ_k are always complex numbers. The meaning of cutoff is then less definite : for instance, a mode can be considered cutoff when $\alpha > \beta$ and propagating when $\alpha < \beta$ the cutoff condition corresponding then to the line $Re(\gamma^2) = 0$ i.e the imaginary axis in the γ^2 plane (other definitions of cutoff corresponding to other contours in the γ^2 or γ plane can also be chosen).

Research of the zeros

The number of modes that can propagate is then given by the number of zeros of $f(\gamma)$ located within a specified area. For the cutoff condition indicated above, this area covers the left-hand half plane. If the structure does not support backward waves, the area can be further limited to the top half of this half-plane ($Re(\gamma^2) < 0, Im(\gamma^2) > 0$) The number N of zeros of the analytical function $f(\gamma)$ located within the contour C in the complex γ plane (or the complex γ^2 plane) is given by the following relation, provided no zero is located directly on the contour C

$$N = \frac{1}{2\pi j} \oint_C \frac{f'(\gamma)}{f(\gamma)} d\gamma \qquad (2)$$

wherethe prime denotes differentiation with respect to γ and the clockwise direction is taken along the contour C. The evaluation of this integral would be rather time consuming; the same information can however be obtained by considering the phase of the function $f(\gamma)$ along the contour, as can be seen by developing (2) :

$$\oint_C \frac{f'(\gamma)}{f(\gamma)} d\gamma = \text{Log } f(\gamma)\Big|_C = \text{Log } \big|f(\gamma)\big|\Big|_C + j\varphi\Big|_C$$

$$= 2\pi j N \qquad (3)$$

On the contour C, the phase of the function $f(\gamma)$ will go through N jumps of 2π Counting the number of jumps allows therefore to determine the number of zeros located within the contour. Since it is impractical to have a contour extending to infinity, a square contour is chosen within the γ^2 plane, delimited by the real and imaginary axes and by parallels to both axes (Fig. 1). This square must be wide enough to contain the zero of the dominant mode. (This can be done by finding a lower bound).

The phase φ defined in the interval $-\pi \leqslant \varphi \leqslant \pi$ is then computed for a number of points located around the contour. N is then equal to the number of jumps from $-\pi$ to $+\pi$ minus the number of jumps from $+\pi$ to $-\pi$ [6,7] (clockwise direction along the contour C).

Knowing the number of roots within the area delimited by the contour C, their exact location can be determined by means of the search method described in [2] (Fig. 2). Suitable starting points for this search process are the points on the contour where jumps of the phase occur.

Conclusion

The developped method allows to determine theoretically the existence of higher-order modes in anisotropic slab-loaded waveguides.

When the propagation constants are known it is possible to calculate simply the electromagnetic field patterns and the power density distribution in the devices. These distributions will allow to determine the mode excitation probabilities and the high fields regions which limit the use of such devices in very high power.

References

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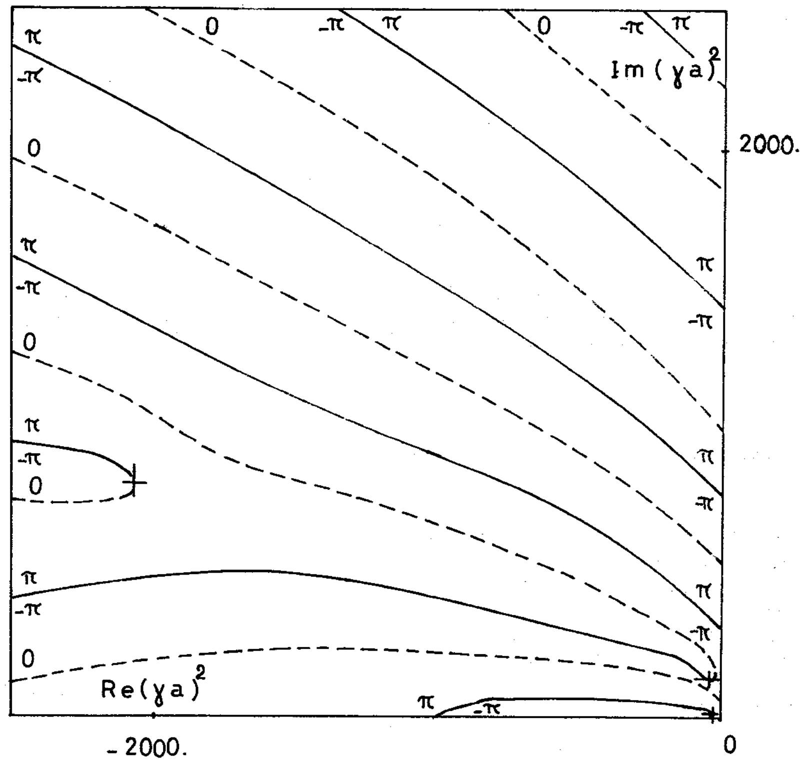

Fig 1 , Equiphase Lines of f(γ) .

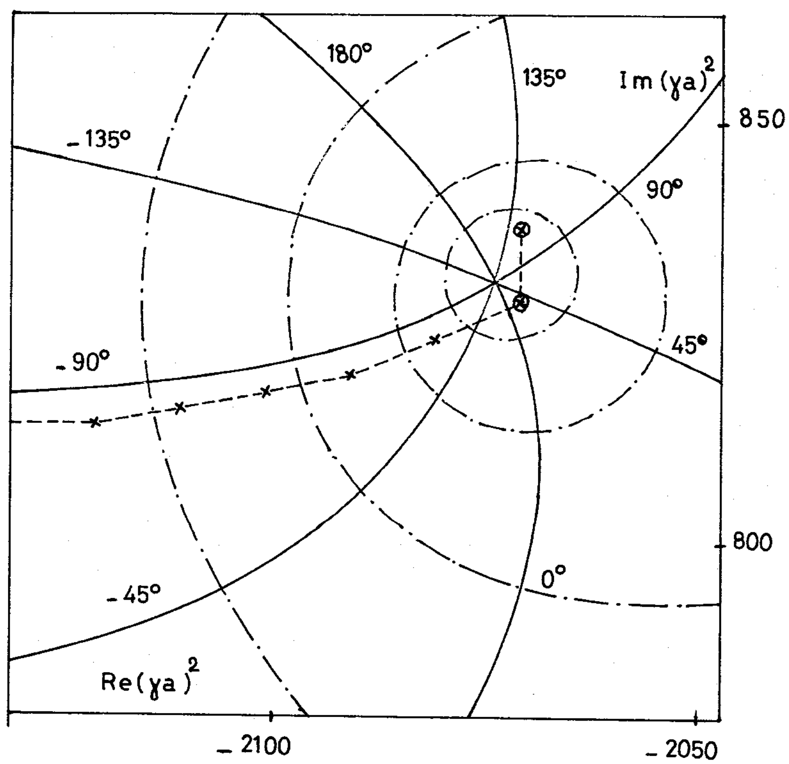

Fig 2 , Research for one zero of f(γ) .